SUPER - TSD

A GENERALIZATION OF THE TSD NETWORK ANALYZER CALIBRATION PROCEDURE, COVERING n - PORT MEASUREMENTS WITH LEAKAGE

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Abstract

The basic philosophy of the THROUGH-SHORT-DELAY calibration procedure for two-port Automated Network Analyzers , has been extended to n-port S-parameter measurements, while also accounting for the possible signal leakage between all port pairs. The system errors are represented by a 2n-port virtual error network, having n ports connected to the device under test (DUT) and n ports connected to an ideal, error-free multiport network analyzer . The $(2n)^2$ T-parameters of the error network are explicitly expressed,in blocks of n^2 at a time,as matricial functions of the 3 n^2 measured S-parameters of three n-port standards, sequentially replacing the DUT during system calibration. Also the possibility has been proved of correcting the errors arising from repeatable port-impedance value-changes, as those found in switching test sets. This capability has been introduced and tested also in the classical two-port TSD algorithm, by means of minor modification and subsequent post-processing .

SUPER - TSD

The THROUGH-SHORT-DELAY (TSD) calibration procedure for two-port Automated Network Analyzers [1] [2],has recently been proved capable of correcting large internal & external errors affecting S-parameter measurements [3]. In particular,non-connectable components,requiring interfacing networks,can be thus accurately measured [4]. Recent theoretical work has extended the capabilities of the TSD method to multiport S-parameter measurements while also accounting for the errors due to all possible signal leakage paths, bypassing the DUT .
No theoretical limitation was found upon the relative amount of signal leakage that can be corrected for .
The basic advantages of the original TSD procedure, represented by the fast execution of the closed-form algorithm and the ability to handle large calibration errors, have been retained in the new, generalized Super-TSD explicit n-port calibration algorithm .
Using rather unconventional matrix algebra operators it has been possible to express the new generalized closed form solution in a concise symbolism, directly translatable in standard programming language .
As the original TSD method,Super-TSD relies on a global representation of all system errors by the scattering response of a virtual error network, connected between the DUT and an ideal error-free multiport network analyzer. For an n-port DUT, the virtual error network is a 2n-port, having n ports connected to it and n ports connected to the multiport network analyzer system .
The most convenient representation of the error network scattering response is given by a complex 2n x 2n T-parameter matrix (Fig. 1). By substituting three n - port standards in place of the object, three pairs of measured S-parameter matrices S_{Mi} and known matrices S_{Si} may be obtained from which matricial expressions of the quadrants T_2 , T_3 and T_4 of the 2n x 2n error network T-matrix can be derived, as functions of the arbitrary matrix quadrant T_1 . The fundamental n x n matricial equations relating the four quadrants to the S_{Mi} , S_{Si} matrix pairs are :

$$\mathbf{S_{Mi}} = (T_1 \bullet S_{Si} + T_2) \bullet (T_3 \bullet S_{Si} + T_4)^{-1} \qquad (1)$$

$$i = 1 , 2 , 3$$

These calibration equations may be rewritten in the form of n^2 x n^2 **matricial equations as :**

$$(I \otimes S_{Si}^T) \cdot RS(T_1) + RS(T_2) - (S_{Mi} \otimes S_{Si}^T) \cdot RS(T_3) - (S_{Mi} \otimes I) \cdot RS(T_4) = 0 \qquad (2)$$

where $A \otimes B$ is the Kroneker Tensor Product [5] of the matrices A and B, I is the n x n unit matrix, A^T is the transpose of matrix A and RS(A) is a " reshuffle " operator which transforms the n x n matrix A into the n^2 - dimensional column vector RS(A),by rotating the rows of A by 90° clockwise and sequentially ordering them in a vertical line.The advantage of form (2) is that all its terms are matrix-by-vector products of the same order n^2 and thus these equations form an homogeneous set of linear, matricial equations in the four vectors $RS(T_i)$, i = 1,2..4 . From this set the following explicit vectorial solution is obtained by Gaussian elimination:

$$RS(T_1) = \text{Arbitrary column vector of order } n^2 \qquad (3)$$

$$RS(T_2) = \Big\{ (S_{M1} \otimes S_{S1}^T)(B^{-1}A - D^{-1}C)^{-1}(B^{-1}E - D^{-1}F) + (S_{M1} \otimes I)(A^{-1}B - C^{-1}D)^{-1}(A^{-1}E - C^{-1}F) - (I \otimes S_{S1}^T) \Big\} \bullet RS(T_1) \qquad (4)$$

$$RS(T_3) = (B^{-1}A - D^{-1}C)^{-1}(B^{-1}E - D^{-1}F) \bullet RS(T_1) \qquad (5)$$

$$RS(T_4) = (A^{-1}B - C^{-1}D)^{-1}(A^{-1}E - C^{-1}F) \bullet RS(T_1) \qquad (6)$$

where :

$$A = (S_{M1} \otimes S_{S1}^T) - (S_{M2} \otimes S_{S2}^T) \qquad (7)$$

$$B = (S_{M1} \otimes I) - (S_{M2} \otimes I) \qquad (8)$$

$$C = (S_{M1} \otimes S_{S1}^T) - (S_{M3} \otimes S_{S3}^T) \qquad (9)$$

$$D = (S_{M1} \otimes I) - (S_{M3} \otimes I) \qquad (10)$$

$$E = (I \otimes S_{S1}^T) - (I \otimes S_{S2}^T) \qquad (11)$$

$$F = (I \otimes S_{S1}^T) - (I \otimes S_{S3}^T) \qquad (12)$$

Once the 2n x 2n T-matrix of the error network has been computed,the removal of all calibration errors from the measured n x n scattering parameter matrix S_M of an unknown n-port network may be performed by computing the " corrected " n x n scattering parameter matrix S_X as :

$$S_X = \Big\{ (T_1 - T_2T_4^{-1}T_3)^{-1} S_M + (T_3 - T_4T_2^{-1}T_1)^{-1} \Big\} \bullet \Big\{ (T_2 - T_1T_3^{-1}T_4)^{-1} S_M + (T_4 - T_3T_1^{-1}T_2)^{-1} \Big\}^{-1} \qquad (13)$$

This matricial "n-Port De-Embedment Equation" has been proved to be invariant to scalar multiplication of the matrix quadrant T_1 by a complex constant and it is expected to be invariant to the arbitrary choice of T_1, although no formal proof of this invariance has yet been given. It is easy to obtain at least three n-port standards with known S-matrices S_{Si}, by using various combinations of THROUGHS , SHORTS and DELAYS .
By using more than three standards , redundancy may be introduced and the possible occurrence of ill-conditioned matrices among those to be inverted may be circumvented . The " reshuffle " operator RS(A) may easily be implemented in FORTRAN by a simple EQUIVALENCE between the matrix A and the vector RS(A),both stored in COMMON.

The described Super-TSD calibration algorithm is expected to be even more effective and usefull than the original TSD,in measurements performed upon multiport subassemblies and systems , especially in the presence of substantial multiple signal leakages bypassing the DUT. Possible examples are : multibeam antenna feeders and multichannel microwave interferometers for thermonuclear plasma diagnostics.

SWITCHING ERRORS

The so-called "switching" S-parameter Test Sets provide speed and convenience in S-parameter acquisition , but introduce additional errors due to possible non-repeatability of the microwave switches and measurement-port impedance changes,reflecting the internal rearrangement of the RF circuit during a full two-port measurement . While switch repeatability is a technological and statistical problem, repeatable port-impedance changes may be considered systematic error sources .
An analysis of these repeatable switching errors in two port measurements has been conducted , by assuming the system to be only affected by repeatable port-impedance changes . Each measurement port was assumed to switch through a series of four arbitrary,complex and frequency dependent impedance values Z_{nij}, according to the following scheme :

Measurement	Impedance Port 1	Impedance Port 2
S_{11}	Z_{111}	Z_{211}
S_{12}	Z_{112}	Z_{212}
S_{21}	Z_{121}	Z_{221}
S_{22}	Z_{122}	Z_{222}

The effects of measurement-port mismatch changes may be simulated, analytically and numerically,through a parameter transformation expressing the ("Super - Generalized") S^*_{ij} - parameters of a network,measured by a switching set,as functions of its standard S_{ij} - parameters (measured in a 50 Ohm system) and of the eight port impedances Z_{nij} . The S - to - S^* transformation is given by :

$$S^*_{11} = \frac{C_{111}X + C_{211}Y + C_{311}U + C_{411}V}{C_{211}X + C_{111}Y + C_{411}U + C_{311}V} \qquad (14)$$

$$S^*_{12} = \frac{4\,S_{12}}{C_{212}X + C_{112}Y + C_{412}U + C_{312}V} \qquad (15)$$

$$S^*_{21} = \frac{4\,S_{21}}{C_{221}X + C_{121}Y + C_{421}U + C_{321}V} \qquad (16)$$

$$S^*_{22} = -\frac{C_{122}X + C_{222}Y - C_{322}U - C_{422}V}{C_{222}X + C_{122}Y + C_{422}U + C_{322}V} \qquad (17)$$

where :

$$X = 1 - DET(S) \quad (18) \qquad U = 1 + DET(S) \quad (19)$$

$$Y = S_{11} - S_{22} \quad (20) \qquad V = S_{11} + S_{22} \quad (21)$$

with : $DET(S) = S_{11}S_{22} - S_{12}S_{21}$ (22)

while the 16 coefficients C_{mij} are expressed by :

$$C_{1ij} = \sqrt{\frac{Z_{2ij}}{Z_{1ij}}} - \sqrt{\frac{Z_{1ij}}{Z_{2ij}}} \qquad (23)$$

$$C_{2ij} = \sqrt{\frac{Z_{2ij}}{Z_{1ij}}} + \sqrt{\frac{Z_{1ij}}{Z_{2ij}}} \qquad (24)$$

$$C_{3ij} = \frac{Z_0}{\sqrt{Z_{1ij}Z_{2ij}}} - \frac{\sqrt{Z_{1ij}Z_{2ij}}}{Z_0} \qquad (25)$$

$$C_{4ij} = \frac{Z_0}{\sqrt{Z_{1ij}Z_{2ij}}} + \frac{\sqrt{Z_{1ij}Z_{2ij}}}{Z_0} \qquad (26)$$

$$C^2_{2ij} - C^2_{1ij} = C^2_{4ij} - C^2_{3ij} = 4 \qquad (27)$$

In the super - generalized S^*_{ij} - matrix each entry is normalized to a different pair of measurement-port impedances Z_{1ij} and Z_{2ij} ,a situation not commonly considered in the existing literature .
Simulated TSD calibration data may be obtained from the S - to - S^* transformation by assuming the port impedances Z_{nij} to be known functions of frequency and by substituting , as standard S-parameters :

For the THROUGH:

$$S_{11T} = S_{22T} = 0 \qquad (28a)$$

$$S_{12T} = S_{21T} = e^{-\rho_1} \qquad (28b)$$

For the SHORT:

$$S_{11S} = S_{22S} = -1 \qquad (29a)$$

$$S_{12S} = S_{21S} = 0 \qquad (29b)$$

For the DELAY:

$$S_{11D} = S_{22D} = 0 \qquad (30a)$$

$$S_{12D} = S_{21D} = e^{-\rho_2} \qquad (30b)$$

An analysis of the reduced expressions of S^*_{12} and S^*_{21} for the THROUGH and the DELAY shows a mutual inconsistency of these data, which can not be accounted for in the original TSD error model,consisting of the two-port Error-Boxes A & B [1] [2] . The inconsistency is due to the determinants of the T-matrices of the THROUGH and the DELAY , expressed by the ratios :

$$DET(T_{D1}) = \frac{S_{12T}}{S_{21T}} \quad (31) \qquad DET(T_{D2}) = \frac{S_{12D}}{S_{21D}} \quad (32)$$

being in general different in the presence of switching errors. The inconsistency can,however,be removed by assuming the impedance-reference line L_2 , used in the DELAY standard, to be "virtually" non-reciprocal with :

$$S_{12} = e^{-\rho_2'} \qquad \text{and} \qquad S_{21} = e^{-\rho_2''}$$

This assumption does not infirm the validity of the line as an impedance standard and does not introduce any new unknowns, as the non-reciprocity is uniquely determined by the known mutual inconsistency of the THROUGH and DELAY data. The S-parameters of the Error-Boxes A & B may still be explicitly computed with the same algorithm , if the fundamental matrix products :

$$H = T_{D1} T_{D2}^{-1} \quad (33) \qquad K = T_{D2}^{-1} T_{D1} \quad (34)$$

are redefined as :

$$H_{NEW} = e^{-\delta} (T_{D1} T_{D2}^{-1}) \quad (35)$$

$$K_{NEW} = e^{-\delta} (T_{D2}^{-1} T_{D1}) \quad (36)$$

where : $e^{-\delta} = \left[\sqrt{DET(H)}\right]^{-1} = \left[\sqrt{DET(K)}\right]^{-1} =$

$$= \sqrt{\frac{S_{12D}}{S_{21D}} \frac{S_{21T}}{S_{12T}}} \quad (37)$$

and : $\delta = 1/2 (\rho_2' - \rho_2'')$ (38)

$\rho_2 = 1/2 (\rho_2' + \rho_2'')$ (39)

A numerical simulation of a Network Analyzer affected only by switching errors has already been performed by assuming the impedances Z_{nij} to be equivalent to eight different resistive mismatches,located at eight different distances from the external measurement-port interfaces,deep inside the switching Test Set . The specific mismatch value and its distance from the port interface have been changed randomly in switching from one S_{ij} measurement to the other, thus introducing 16 randomly chosen parameters in the Z_{nij} impedances of the ports . These impedances also change with frequency in mutually uncorrelated ways .

By numerically computing the S^*_{ij} scattering parameters of the THROUGH , SHORT and DELAY with the expressions (14) - (17) and using these simulated calibration data as input to the TSD Error-Computation-Program , Error-Box solutions have been computed , which appear to represent at least part of the switching error due to the repeatable port-impedance changes (Figrs. 2 - 5) . Indeed , by "stripping" (or deconvolving) the obtained Error-Boxes A & B from the SHORT data we obtain a residual network having $S_{11} = S_{22} = -1$ and $S_{12} = S_{21} = 0$, equivalent to an immediate short at both measurement interfaces.By stripping the Error-Boxes A & B from the THROUGH and the DELAY residual networks are obtained having $S_{11} = S_{22} = 0$ (Fig. 6) to within the rounding-off errors of the processor. In transmission , however, both the THROUGH- and the DELAY-residual-networks show residual magnitude and phase ripples around the expected values of S_{12} and S_{21} of the lines L_1 and L_2 [1] .

Our present conclusion is that , although the two-port TSD Error-Box Model is fully capable of representing any errors other than switching, including any external interfacing network,it can not in general represent the totality of the switching errors.In situations compounding switching and non-switching errors , the stripping of the computed Error-Boxes will remove the totality of the non-switching errors and an unspecified part of the switching errors .

We assume that the residual error,after stripping,behaves as a "residual switching" and can be represented by an S - to - S^* transformation as defined by (14) - (17), based on "equivalent" port-impedances Z_{nij} and corresponding C_{mij} coefficients. A method has been developed for computing the equivalent C_{mij} coefficients of the residual switching error,from the scattering parameters of the "Residual-THROUGH" , of the "Residual-DELAY" and those of an auxiliary reciprocal , non-symmetric network , to be measured as a reference object in forward and backward insertion , during calibration .

The obtained C_{mij} coefficients are then used to post-process measured data for final correction , after the stripping of the Error-Boxes . The code implementing these operations is being debugged at this writing .

CONCLUSION

A new calibration procedure has been developed extending the basic philosophy of TSD to multiport scattering parameter measurements,affected by multiple signal leakage. The new Super-TSD procedure computes a global error representation by means of explicit matricial expressions using the measured scattering parameters of at least 3 multiport standards.Programs implementing the Super-TSD algorithm are being written .

A study has also been conducted of the capability of the classical two-port TSD to correct "switching errors" due to repeatable measurement-port mismatch changes typical of switching S-Parameter Test Sets. It appears that TSD can be made to correct for these errors too,by means of a minor modification and added data post-processing to be applied after "de-embedment" or "stripping of the Error-Boxes A & B" from the uncalibrated measurements .

REFERENCES

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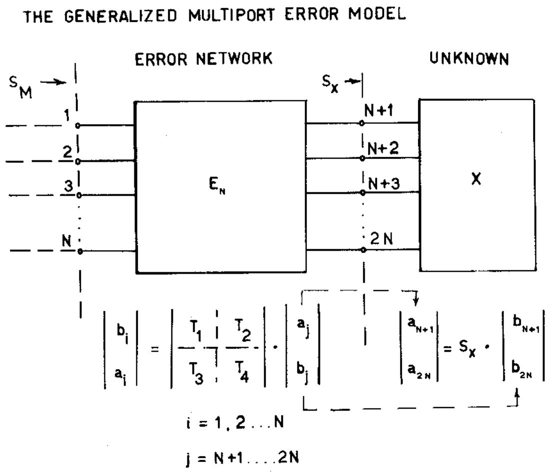

Fig. 1 - The Super-TSD Error Model is a 2n-Port Virtual Error Network interfacing the Unknown n - Port to an ideal n - Port Network Analyzer .

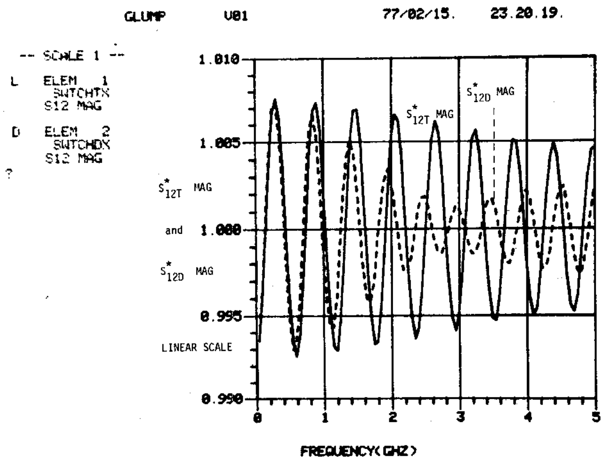

Fig. 2 - Apparent backward transmission of the THROUGH and DELAY in presence of repeatable switching errors due to frequency dependent port-mismatch.

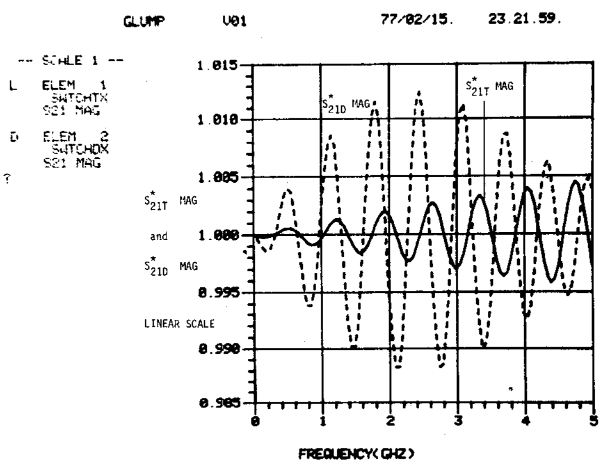

Fig. 3 - Apparent forward transmission of the THROUGH and DELAY in presence of switching errors . Comparison with Fig. 2 shows different S_{12}/S_{21}.

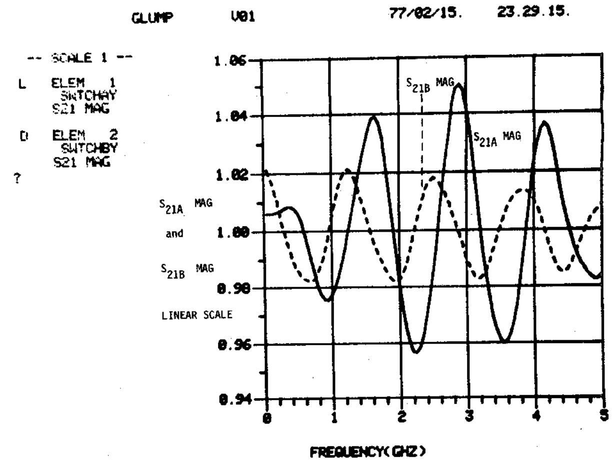

Fig. 4 - Magnitude of the forward transmission for the Error-Boxes A & B computed from simulated TSD calibration data representing switching errors.

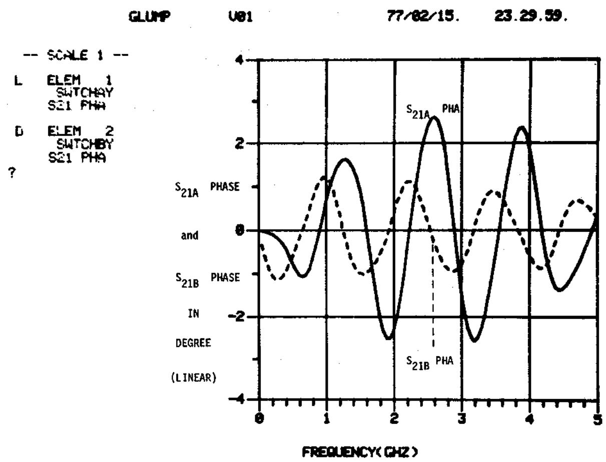

Fig. 5 - Phase of the forward transmission for the Error-Boxes A & B computed from simulated TSD calibration data representing switching errors.

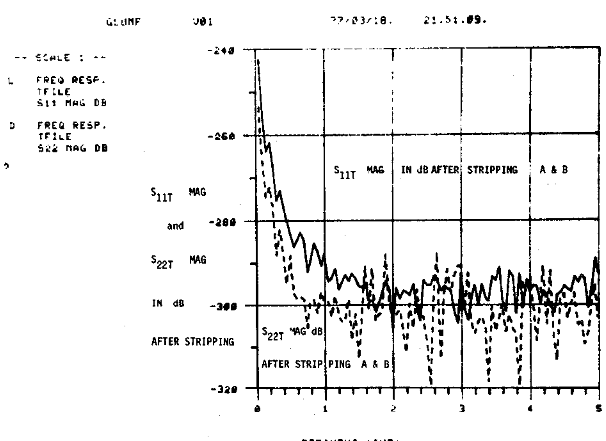

Fig. 6 - The residual network obtained by stripping the Error-Boxes from the THROUGH data appears to be matched to within the rounding-off errors .